Reaction-diffusion equations in biology

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Reaction-diffusion equation (parabolic PDE)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + F(u)$$

heat or mass production

mass diffusion or heat conduction

Outline

- **Lecture 1: Examples of biological phenomena and methods of modelling**
- **Lecture 2: Reaction-diffusion waves**
- **Lecture 3: Stability and pattern formation**

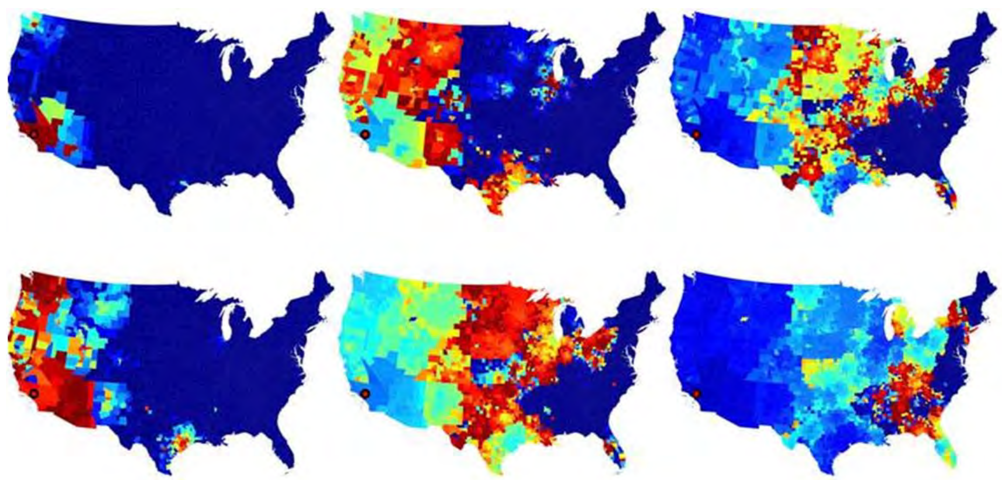
Lecture 1:
Examples of biological phenomena and methods of modelling

Examples of biological phenomena

- Ecology and evolution
- Morphogenesis and pattern formation
- Modelling in physiology

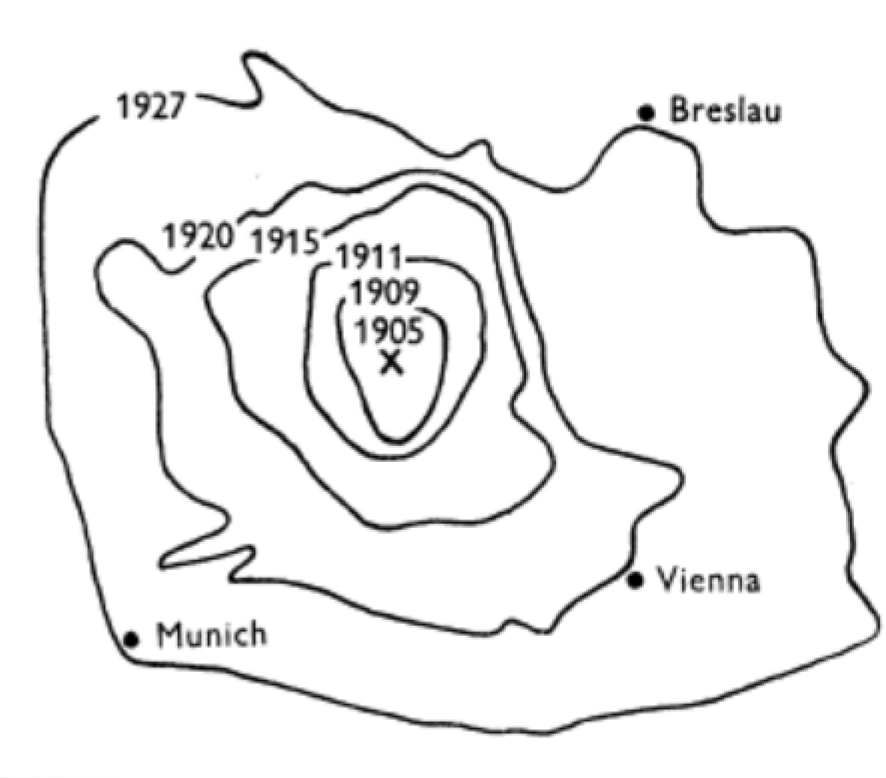
Ecology and evolution

Ecological invasions



Infection spreading in the USA (model: epidemiology, SIR)

Invasion of muskrats



Skellam, 1951

Model: KPP

THE ORIGIN OF SPECIES

BY MEANS OF NATURAL SELECTION,

OR THE

PRESERVATION OF FAVOURED RACES IN THE STRUGGLE FOR LIFE.

BY CHARLES DARWIN, M.A.,

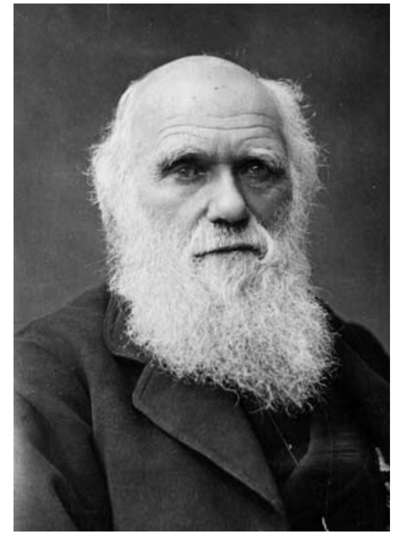
FELLOW OF THE ROYAL, GEOLOGICAL, LINNÆAN, ETC., SOCIETIES;

AUTHOR OF 'JOURNAL OF RESEARCHES DURING H. M. S. BEAGLE'S VOYAGE ROUND THE WORLD.'

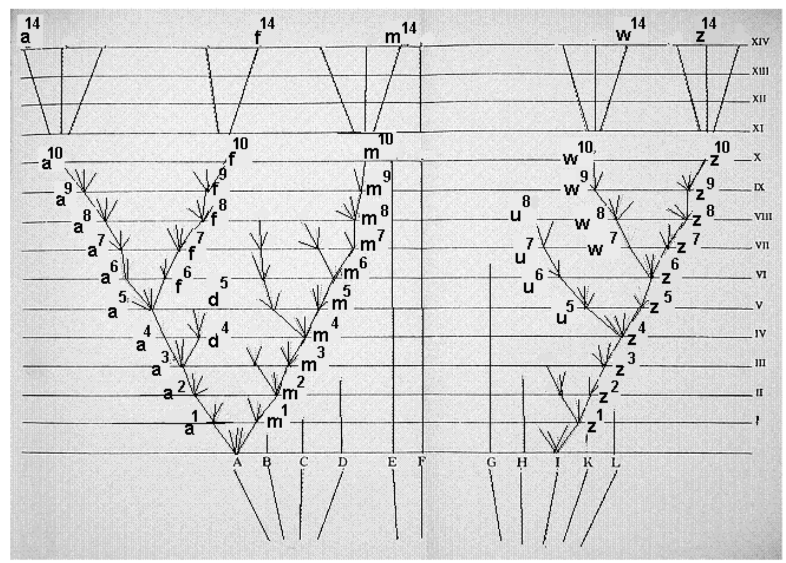
LONDON:

JOHN MURRAY, ALBEMARLE STREET.

1859.



1809-1882



Model: nonlocal reaction-diffusion eq

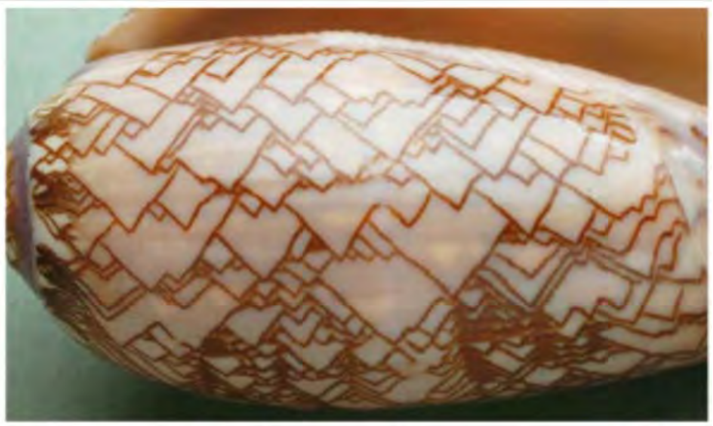
Morphogenesis and pattern formation

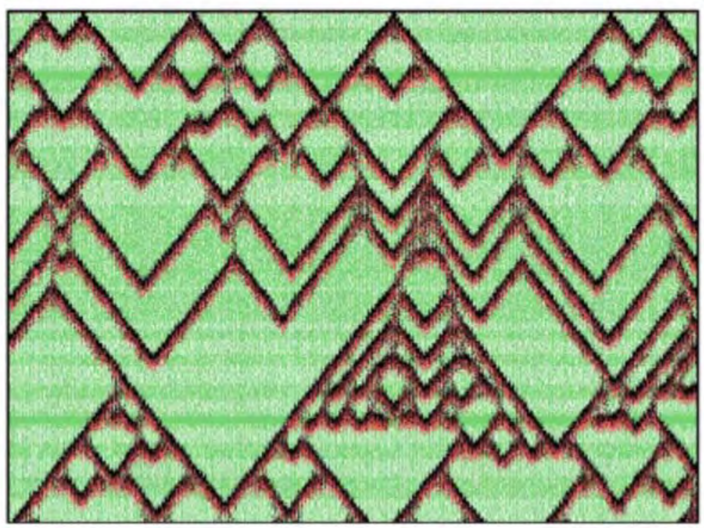
Morphogenesis and biological pattern formation

Model: reaction-diffusion systems
(Turing instability)

See shells (H. Meinhardt)





Branching patterns in plants

$$\frac{\partial C}{\partial t} + V\frac{\partial C}{\partial x} = d_C\frac{\partial^2 C}{\partial x^2} - \gamma C$$

$$\frac{\partial K}{\partial t} - V_K\frac{\partial K}{\partial x} = d_K\frac{\partial^2 K}{\partial x^2} - \mu K$$

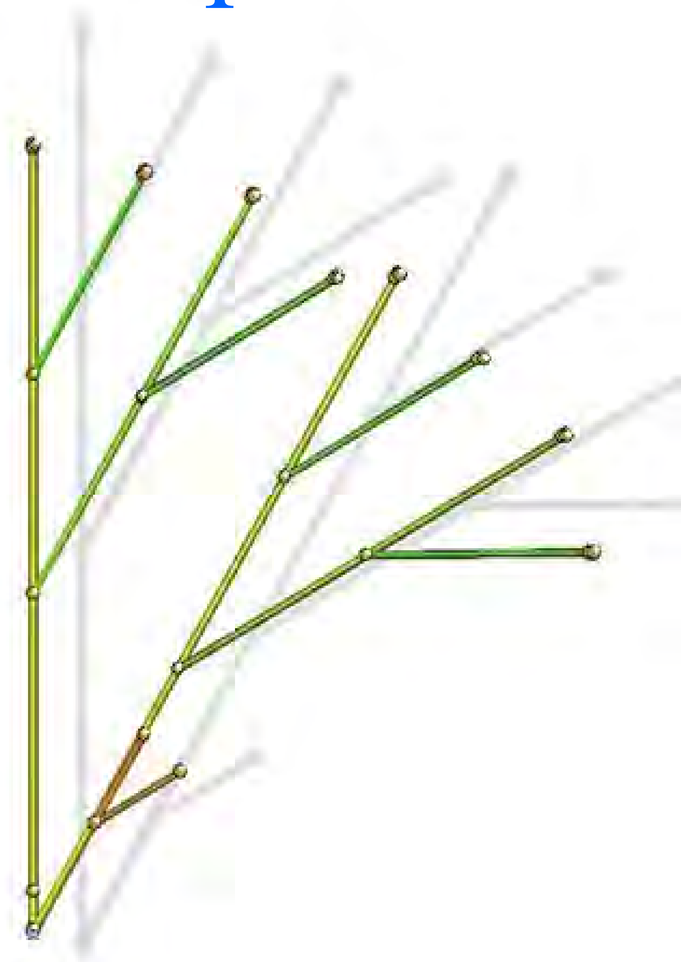
$$\frac{\partial A}{\partial t} - V_A\frac{\partial A}{\partial x} = d_A\frac{\partial^2 A}{\partial x^2} - \nu A$$

$$\frac{dL}{dt} = V, \quad V = f(R)$$

$$h\frac{dR}{dt} = F(A, K)g(R)C - \sigma R$$

$$F(A, K) = F_1(A)F_2(K)$$

$$F_1(A) = \begin{cases} kA, & 0 \le A \le 1/k, \\ 1, & 1/k \le A \le A_0 - 1/k, \\ k(A_0 - A), & A_0 - 1/k \le A \le A_0, \\ 0, & A_0 \le A \end{cases}$$



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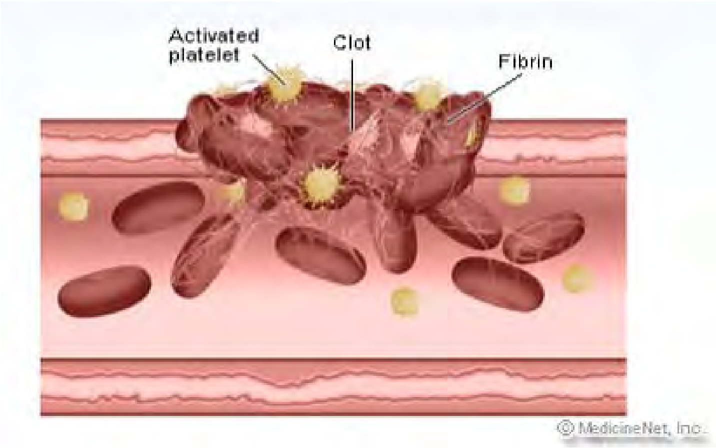
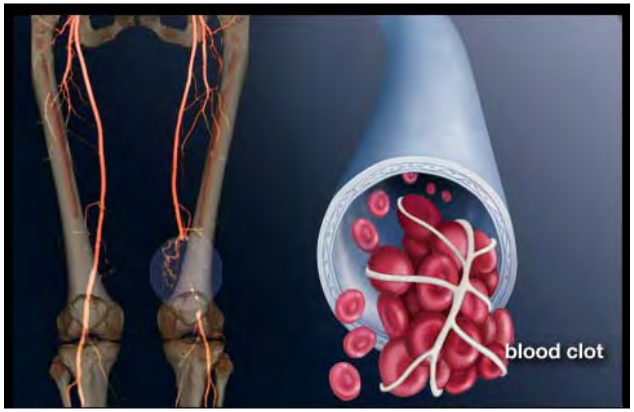
Biomedical applications

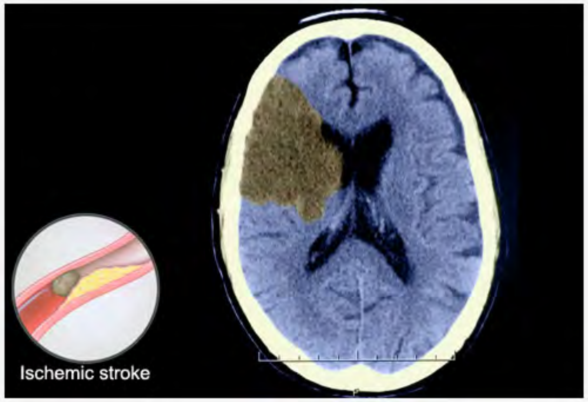
Blood coagulation and related pathologies



Clot growth



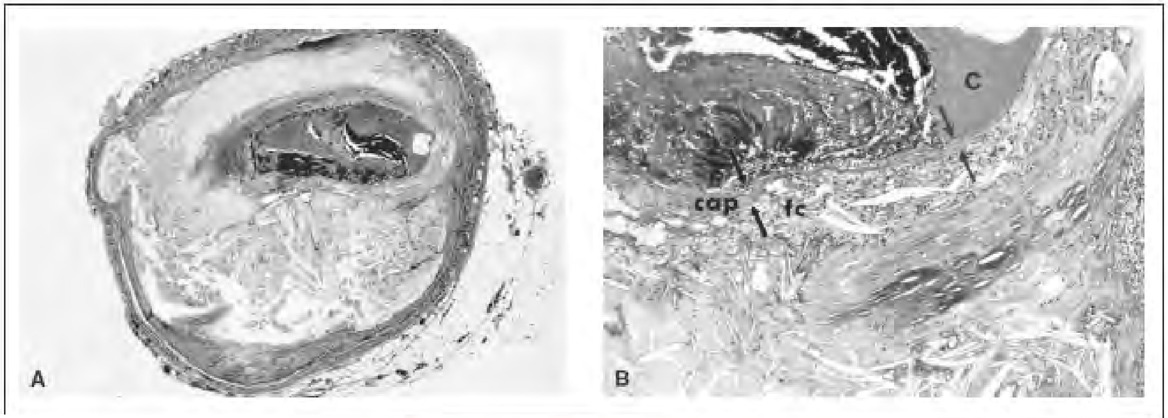
Deep vein thrombosis

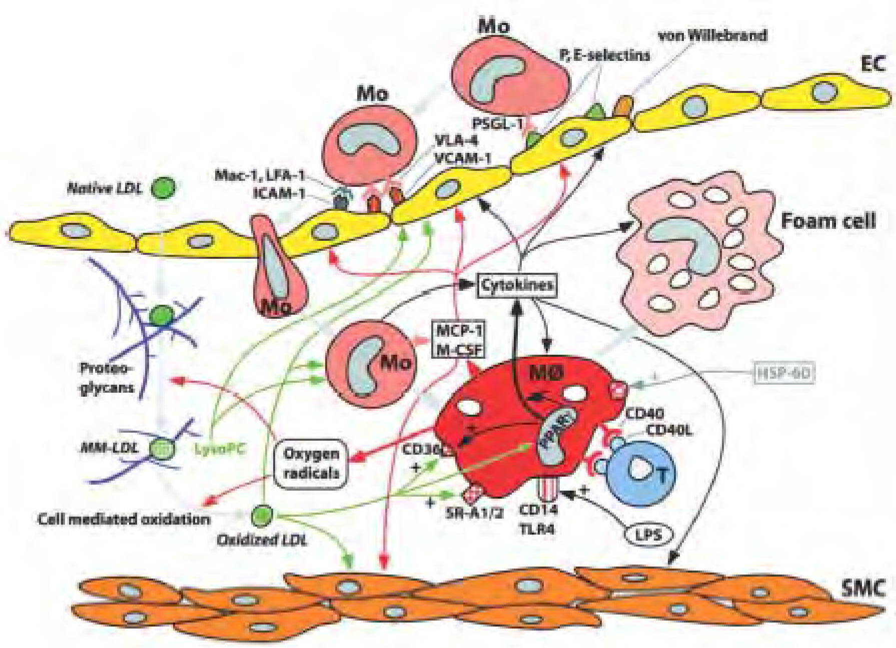


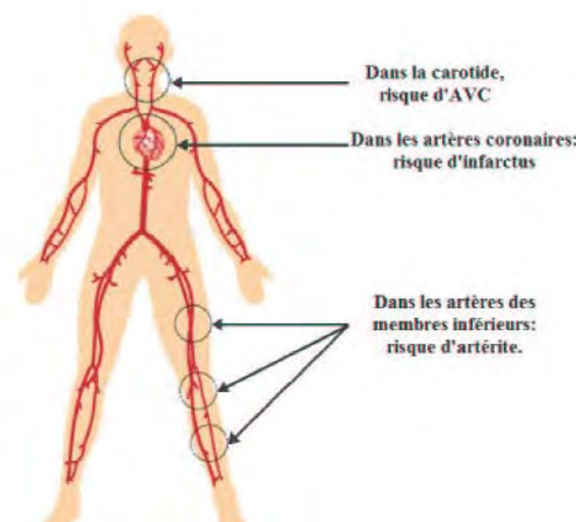
The most common type of stroke is known as an ischemic stroke. Nearly nine out of 10 strokes fall into this category. The culprit is a blood clot that obstructs a blood vessel inside the brain. The clot may develop on the spot or travel through the blood from elsewhere in the body.

Model: RDS + NS

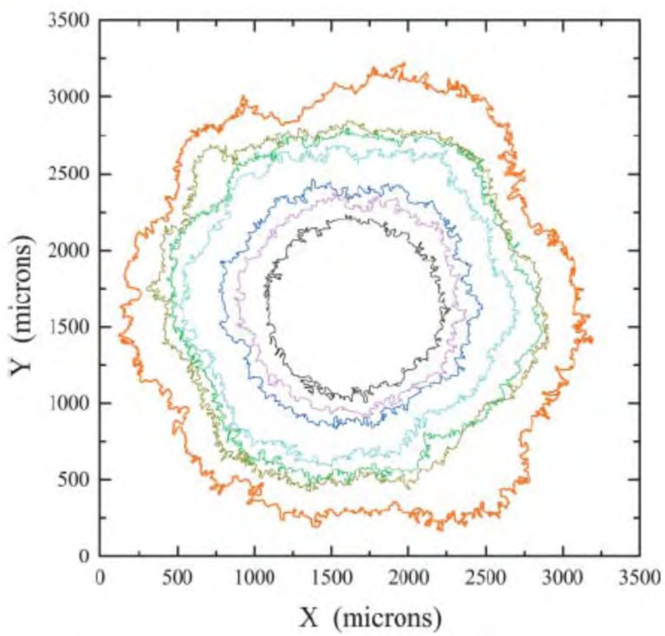
Atherosclerosis

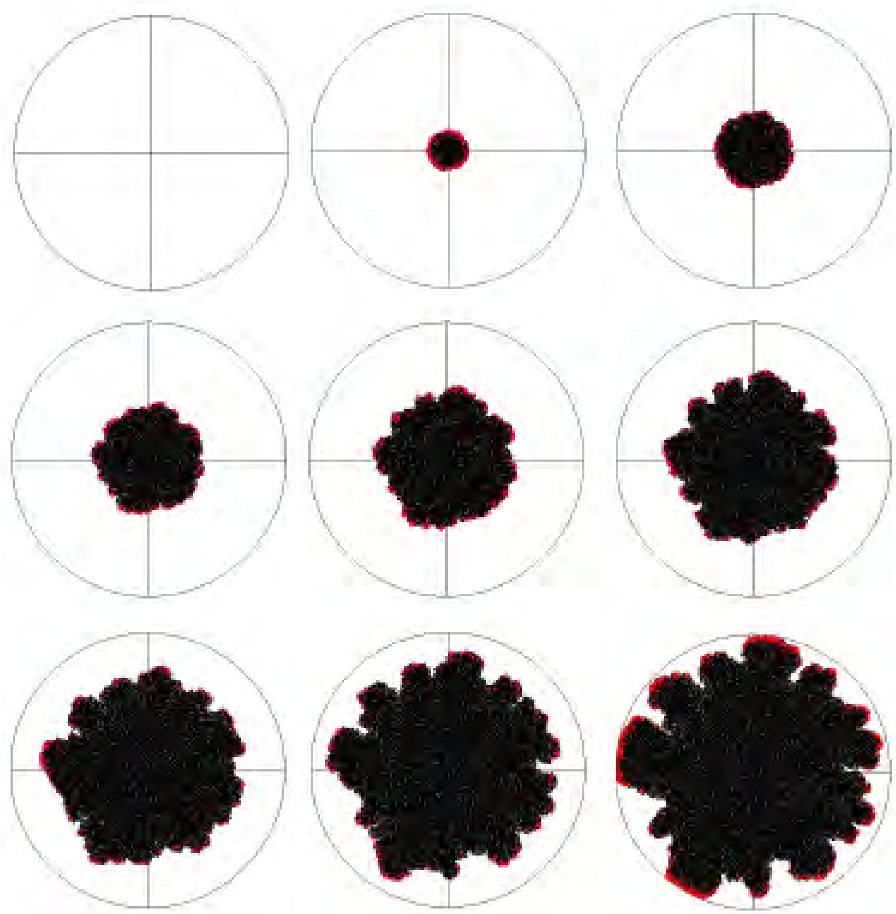






Tumor growth





$$\frac{\partial u}{\partial t} = d\,\frac{\partial^2 u}{\partial x^2} + au(1-u)$$

Methods of modelling

- Ordinary differential equations
- Partial differential equations
- Hybrid (discrete-continuous) models: individual based, ODEs, PDEs
- …

Ordinary differential equations

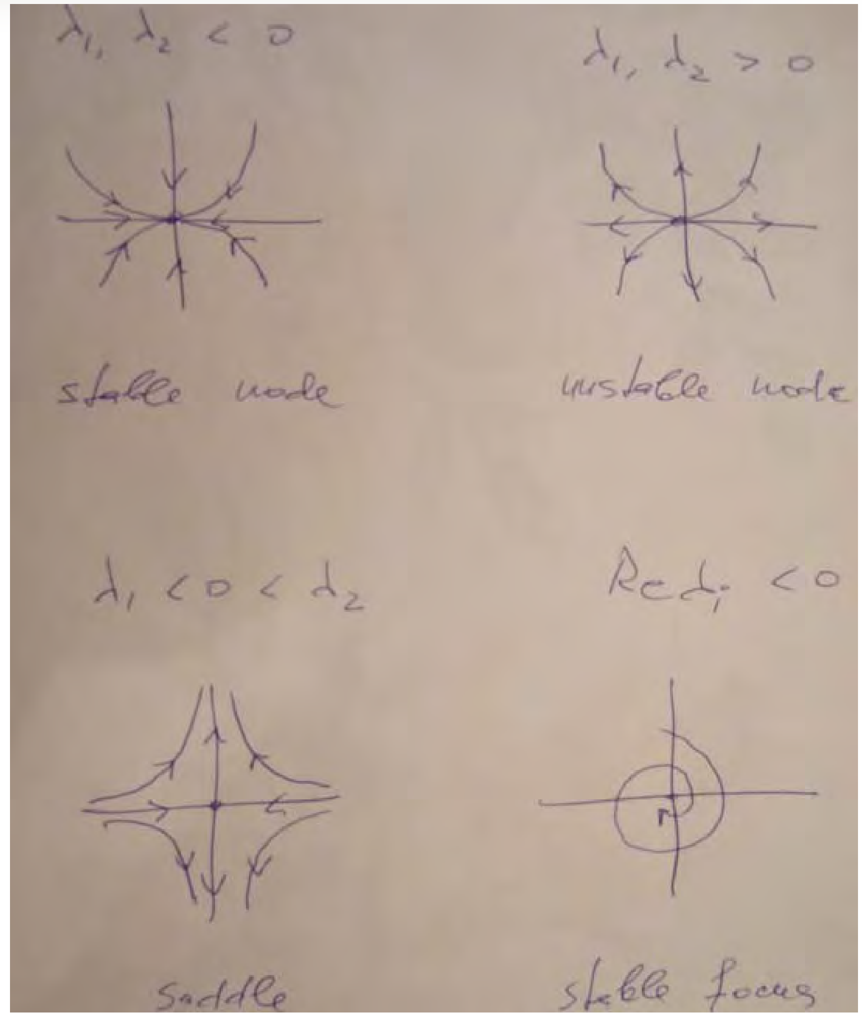
$$\begin{cases} \dfrac{du}{dt} = F(u, v) \\ \dfrac{dv}{dt} = G(u, v) \end{cases}$$

$$F(0,0) = G(0,0) = 0$$

Stability of stationary points

$$A = \begin{pmatrix} F'_u(0,0) & F'_v(0,0) \\ G'_u(0,0) & G'_v(0,0) \end{pmatrix}$$

$$\det(A - \lambda E) = 0$$

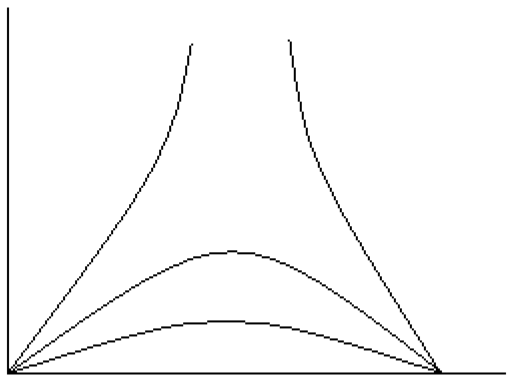


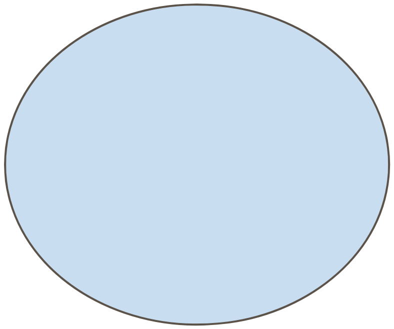
Reaction-diffusion equations

Heat explosion

Temperature distribution

$$\frac{\partial u}{\partial t} = \Delta u + e^{u}$$

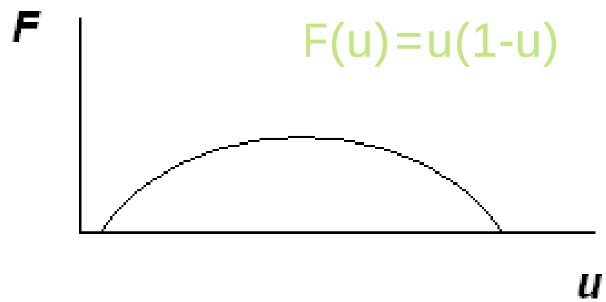


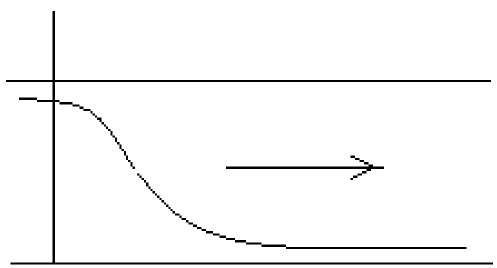


boundary condition: u=0

Propagation of waves

$$\frac{\partial u}{\partial t} = d\,\frac{\partial^2 u}{\partial x^2} + F(u)$$





Existence for all speeds > or = minimal velocity

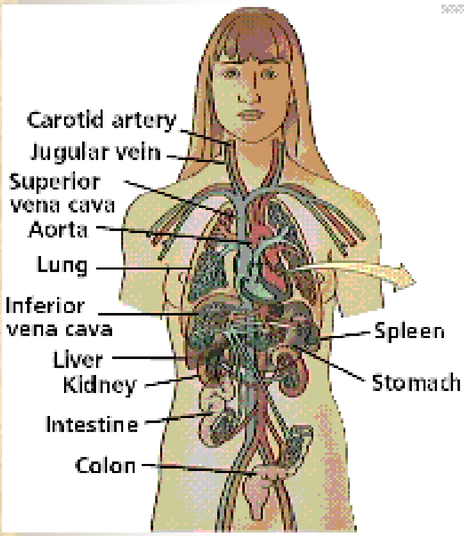
Global convergence to waves

$u(x,t) = w(x-ct)$

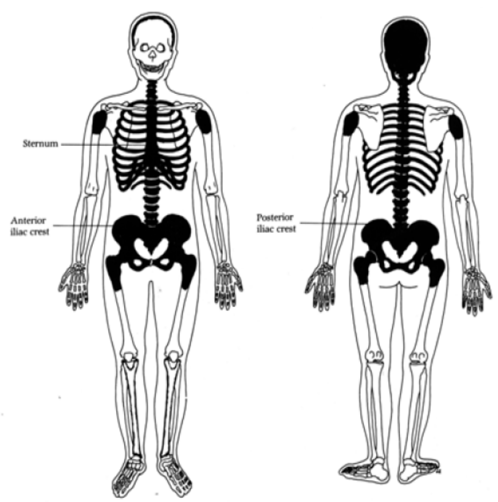
$w'' + c\,w' + F(w) = 0$

Hybrid and multi-scale models

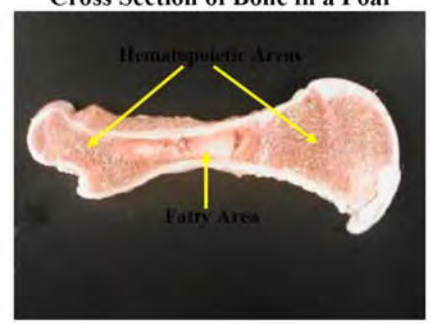
Multi-scale modelling in biology and medicine

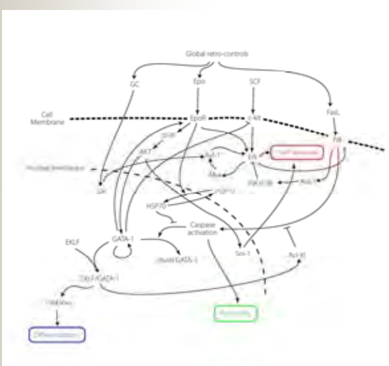


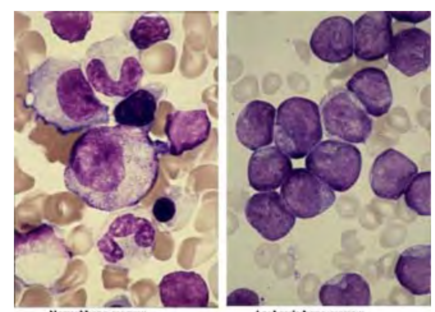
Anatomical Sites of Hematopoiesis in Adult Humans



Cross Section of Bone in a Foal

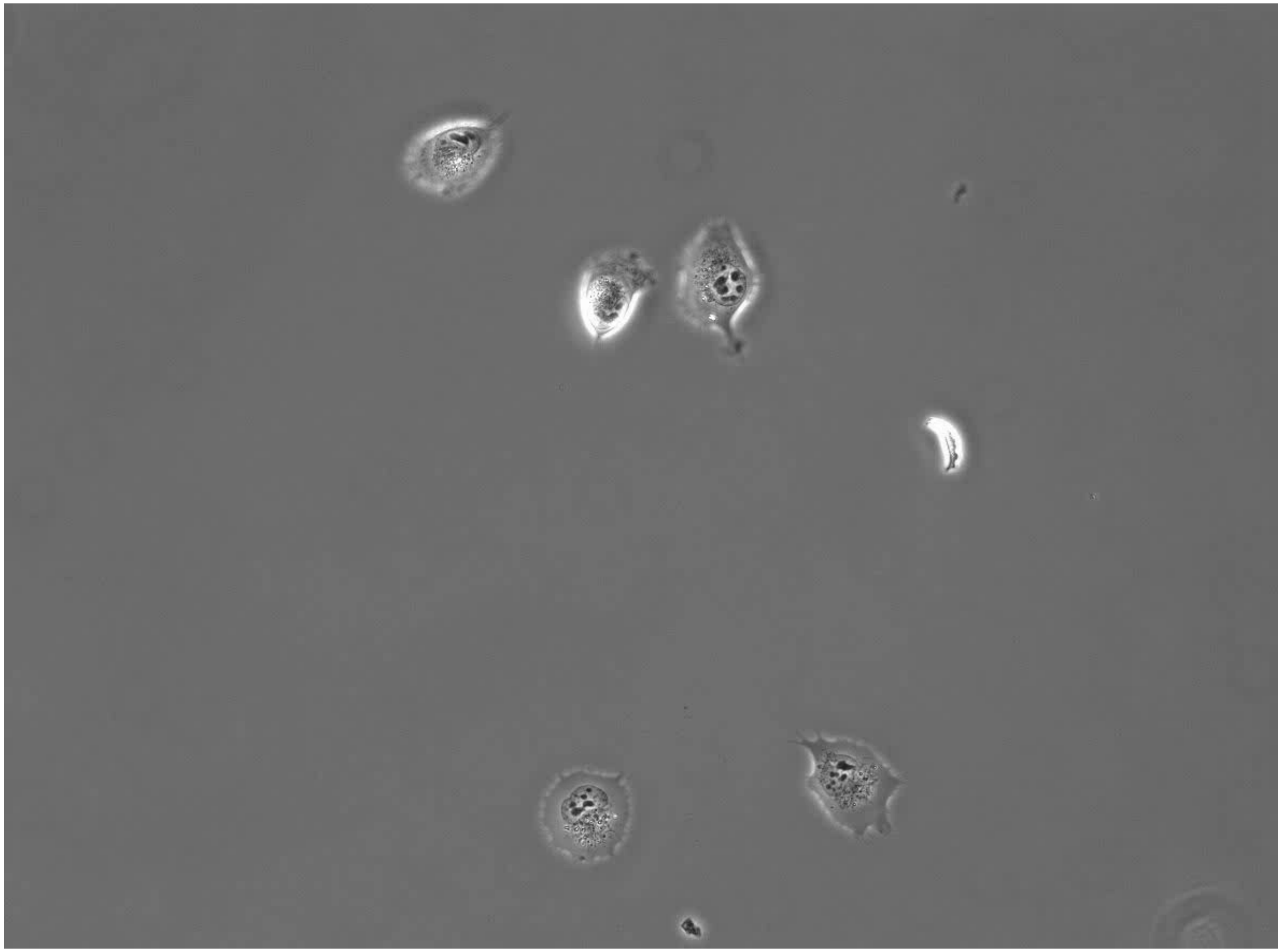




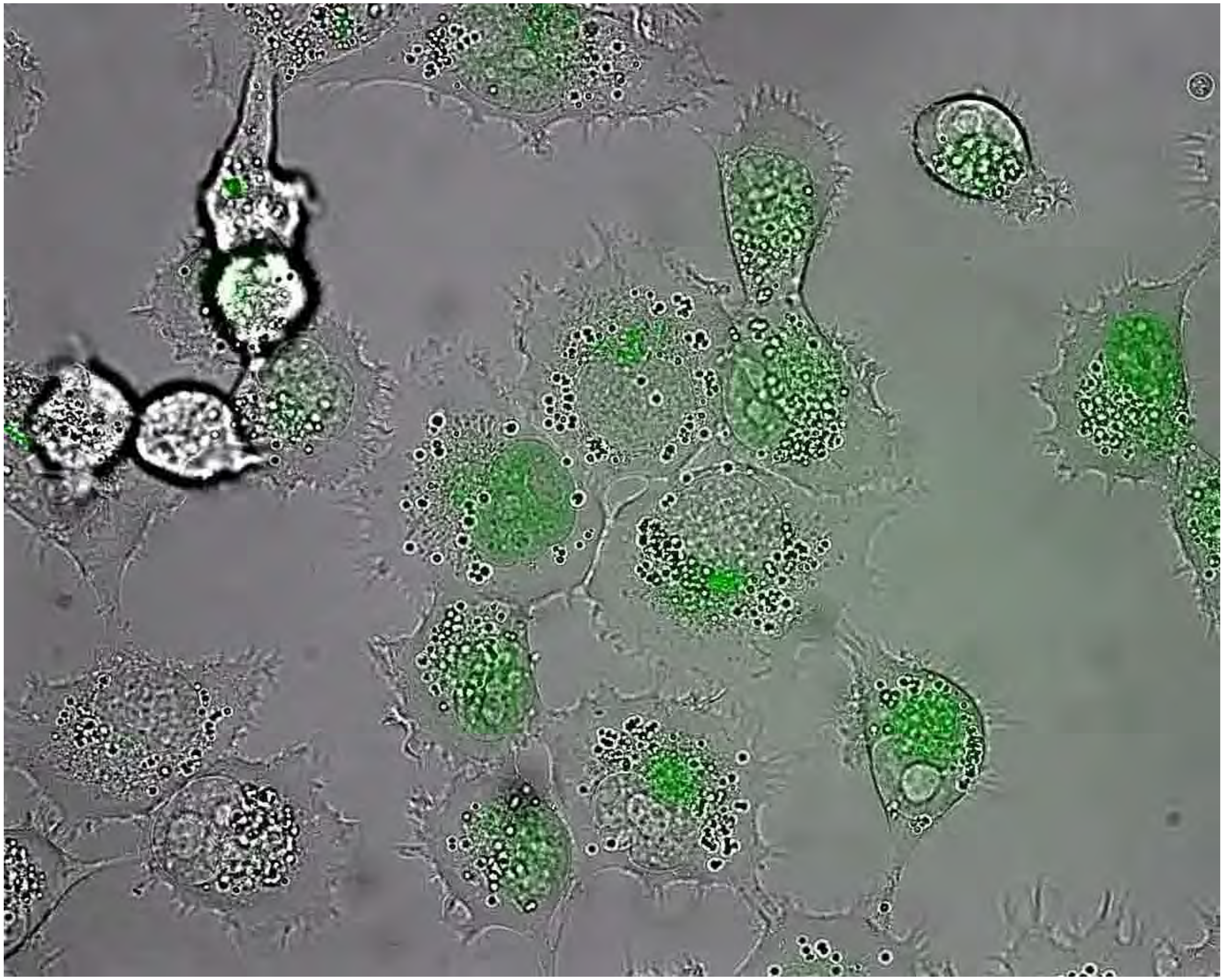


Normal bone marrow Leukemic bone marrow

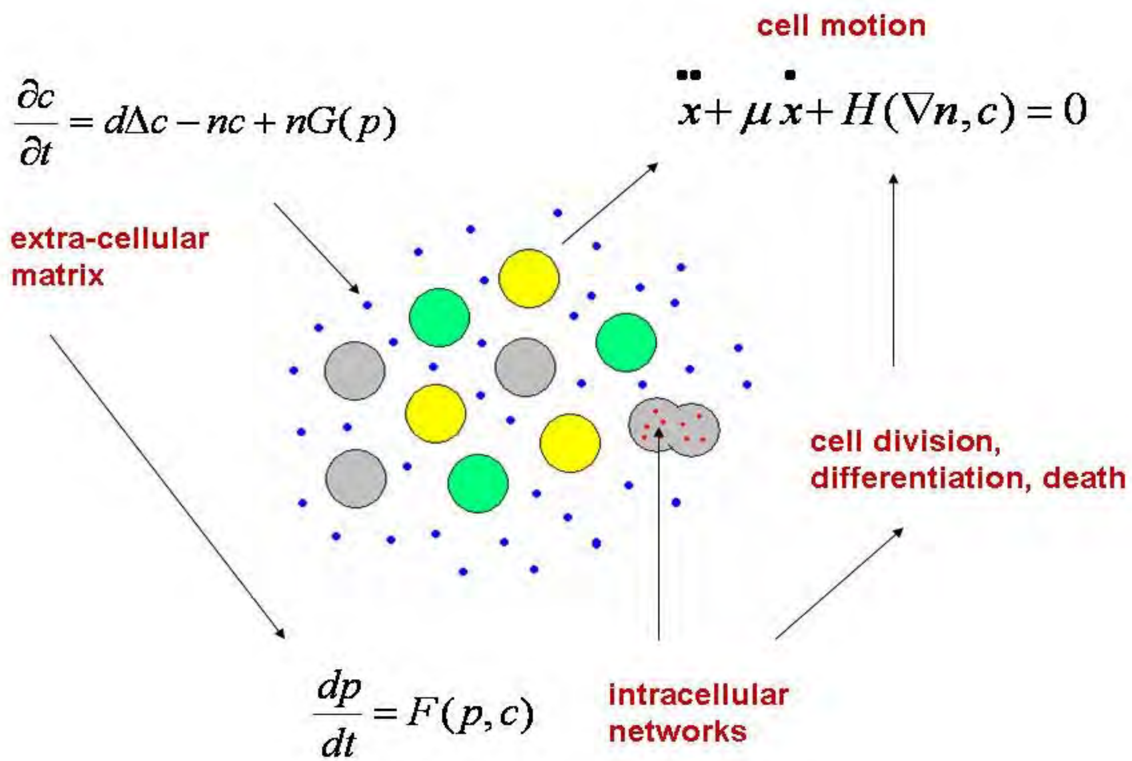
Cell motion and division



Apoptosis



Hybrid models



Erythropoiesis